Measurement Uncertainty in Network Analyzers: Differential Error Analysis of Error Models Part 4: Non-Zero Length Through in Full Two-Port SLOT Calibration

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Abstract

The most accurate full two-port calibration of a VNA Vector Network Analyzer requires a Direct or Zero–Length Through connection. However, it is not uncommon at all to have one or two cables and a DUT Device Under Test with incompatible connectors, either of different type or of the same type/sex, which enforce then the use of some kind of barrel or adapter. Thus, in this paper, we study these cases of Indirect or Non-Zero Length Through, we estimate the effects of such connections on the measurement uncertainty by using our theory of Differential Error Regions and Intervals DERs/DEIs, and we evaluate our resulting method by applying it in practice to a built two–port network, which was measured against frequency with a SLOT calibrated VNA extended by two lengthy cables.

Keywords

microwave measurements, network analyzer, differential error region, differential error interval, calibration

Introduction

A full two-port calibration for the measurement of a two port DUT (Device Under Test) or AUT (Antenna Under Test) with a Vector Network Analyzer involves the well-known two-port error model shown in Fig. 1, for both the forward and reverse directions. The notation of one symbol per system error was adopted in order to keep the rather lengthy expressions as simple as possible. So, D is the Directivity error E_D, M the Source Match error E_M or E_S, R the frequency Response error E_R, L the Load Match error E_L, T the transmission Tracking error E_T, X the isolation or Crosstalk error E_X, with the primed symbols for the reverse direction and m_{ij} the measurements of the DUT. Thus, 12 System Errors and 4 unknowns, the S-parameters of the DUT or AUT must be determined. In [1], [2] the most accurate case of a Direct Zero-Length through, often referenced as "Thru", was examined and presented in detail.

However, there are at least two cases where a Zero-Length Through is impossible and a Non-Zero Length Through is inevitable. First, when the DUT or the cables has the same connectors on each port both in type and sex, and second, when its ports have different type of connector. The Through Standard can be represented then, in general, by the two-port flow graph shown at Fig. 2, where the symbols T_{ij} were used for its S-parameters.

The first announcement of the present work was a twenty minute presentation in the 32nd ANAMET meeting of the National Physical Laboratory (NPL) in 16 October 2009 in Teddington, which is available either in

"http://resource.npl.co.uk/docs/networks/anamet/members_only/meetings/32/20091016_anamet32_thrace.pdf"

or from

www.antennas.gr/anamet/32/

Research

The typical differential errors dS_{11} and dS_{21} of the S_{11} and S_{21} parameters are given by (1) and (2) respectively [1] where (3) is their common denominator P. The expressions are general, and are simplified a lot when the SLO calibration standards are considered. What it is really changed with the Indirect Through connection between the two ports is the L and T system errors shown with blue color and their differential errors dL and dT shown with red color, for the forward and the reverse direction.

The full expressions of these quantities are given in (4)-(8) where the red characters in L, T expressions indicate what is added when the case of the Non-Zero Length Through is considered in comparison with the Direct Through case represented by black characters. At the dL, dT expressions four (4) more terms occur which are depended on the differential errors dT_{ij} of the Through standard. These relations result from the equivalent system of Fig. 1 when the DUT is substituted with the Through represented by the known two port of Fig. 2 and m_{ij} with the corresponding t_{ij} measurements.

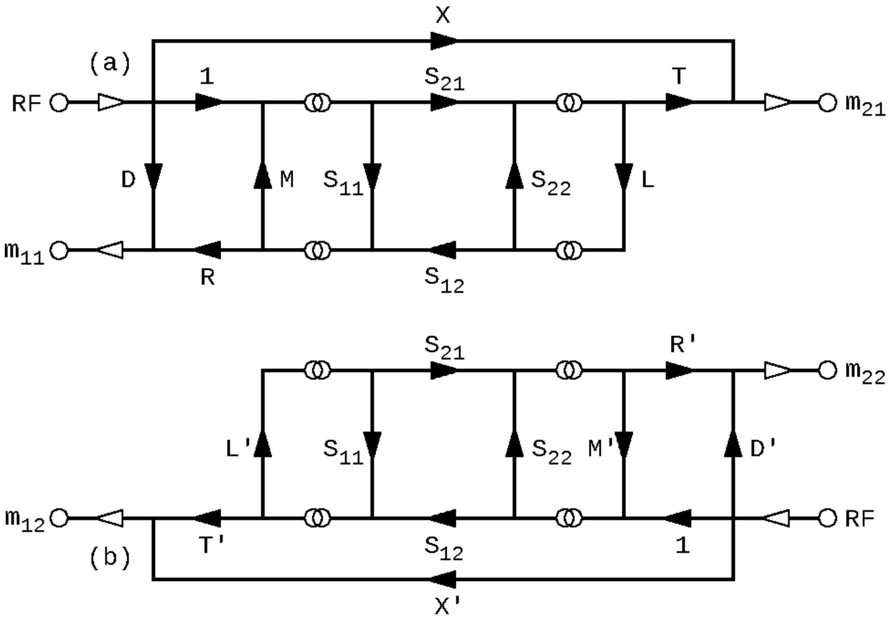

Fig. 1: Two-Port Error Model: Forward and Reverse direction

$$
\begin{aligned}
dS_{11} = \{ & T\,T'(1 - MS_{11})[R' + M'(m_{22} - D')](dm_{11} - dD) \\
& - RR'L(1 - L'S_{11})[(m_{21} - X)(dm_{12} - dX') + (m_{12} - X')(dm_{21} - dX)] \\
& + M'T\,T'[(m_{11} - D)(1 - MS_{11}) - RS_{11}](dm_{22} - dD') \\
& - T\,T'S_{11}(m_{11} - D)[R' + M'(m_{22} - D')]dM \\
& + T\,T'(m_{22} - D')[(m_{11} - D)(1 - MS_{11}) - RS_{11}]dM' \\
& - (R'L(1 - L'S_{11})(m_{12} - X')(m_{21} - X) \\
& + T\,T'S_{11}[R' + M'(m_{22} - D')])dR \\
& - (RL(1 - L'S_{11})(m_{12} - X')(m_{21} - X) \\
& - T\,T'[(m_{11} - D)(1 - MS_{11}) - RS_{11}])dR' \\
& - RR'(m_{12} - X')(m_{21} - X)[(1 - L'S_{11})dL - LS_{11}dL'] \\
& + [(m_{11} - D)(1 - MS_{11}) - RS_{11}][R' + M'(m_{22} - D')] \\
& \cdot (T'dT + TdT')\}/P
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
dS_{21} = \{ & - MT\,T'S_{21}[R' + M'(m_{22} - D')](dm_{11} - dD) \\
& + RR'LL'S_{21}(m_{21} - X)(dm_{12} - dX')
\end{aligned}
$$

$$+ R\{T'[R' + (m_{22} - D')(M' - L)] + R'LL'S_{21}(m_{12} - X')\}(dm_{21} - dX)$$
$$+ T'(R(m_{21} - X)(M' - L) - M'TS_{21}[R + M(m_{11} - D)])(dm_{22} - dD')$$
$$- T\,T'S_{21}(m_{11} - D)[R' + M'(m_{22} - D')]dM$$
$$+ T'(m_{22} - D')(R(m_{21} - X) - TS_{21}[R + M(m_{11} - D)])dM'$$
$$+ \{(m_{21} - X)(T'(m_{22} - D')(M' - L) + R'[T' + LL'S_{21}(m_{12} - X')])$$
$$- T\,T'S_{21}[R' + M'(m_{22} - D')]\}dR$$
$$+ (R(m_{21} - X)[T' + LL'S_{21}(m_{12} - X')]$$
$$- T\,T'S_{21}[R + M(m_{11} - D)])dR'$$
$$+ R(m_{21} - X)[R'L'S_{21}(m_{12} - X') - T'(m_{22} - D')]dL$$
$$+ RR'LS_{21}(m_{12} - X')(m_{21} - X)dL'$$
$$- T'S_{21}[R + M(m_{11} - D)][R' + M'(m_{22} - D')]dT$$
$$+ (R(m_{21} - X)[R' + (m_{22} - D')(M' - L)]$$
$$- TS_{21}[R + M(m_{11} - D)][R' + M'(m_{22} - D')])dT'\}/P \quad (2)$$

$$P = T\,T'[R' + M'(m_{22} - D')][R + M(m_{11} - D)]$$
$$- RR'LL'(m_{12} - X')(m_{21} - X) \quad (3)$$

$$L = \{T_{11}[R + M(t_{11} - D)] - (t_{11} - D)\}$$
$$/ \{[R + M(t_{11} - D)]\Delta - (t_{11} - D)T_{22}\} \quad (4)$$

$$T = -T_{12}(t_{21} - X)R/\{[R + M(t_{11} - D)]\Delta - (t_{11} - D)T_{22}\} \quad (5)$$

$$dL = \{[(1 - MT_{11}) + L(M\Delta - T_{22})(dD - dt_{11}) + (t_{11} - D)(T_{11} - L\Delta)dM$$
$$+ (T_{11} + L\Delta)dR + [R + M(t_{11} - D)](1 - LT_{22})dT_{11}$$
$$+ L\{(t_{11} - D) - T_{11}[R + M(t_{11} - D)]\}dT_{22}$$
$$+ L[R + M(t_{11} - D)](T_{21}dT_{12} + T_{12}dT_{21})\}$$
$$/ \{[R + M(t_{11} - D)]\Delta - (t_{11} - D)T_{22}\} \quad (6)$$

$$dT = \{T(M\Delta - T_{22})(dD - dt_{11}) - T\Delta(t_{11} - D)dM - [T\Delta + (t_{21} - X)T_{12}]dR$$
$$+ T[R + M(t_{11} - D)](T_{12}dT_{21} - T_{22}dT_{11})$$
$$- \{R(t_{21} - X) - T[R + M(t_{11} - D)]T_{21}\}dT_{12}$$
$$- T\{T_{11}[R + M(t_{11} - D)] - (t_{11} - D)\}dT_{22} - RT_{12}(dt_{21} - dX)\}$$
$$/ \{[R + M(t_{11} - D)]\Delta - (t_{11} - D)T_{22}\} \quad (7)$$

$$\Delta \equiv T_{11}T_{22} - T_{12}T_{21} \quad (8)$$

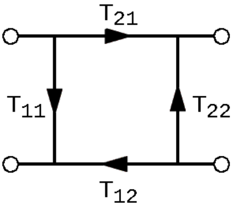

Fig. 2: Non-Zero Length Through

Following the same procedure as in [1], for the final expressions of L, T, dL, dT and the corresponding primed ones L', T', dL', dT' for the reverse direction, as functions of the differentials for standard uncertainties dA, dB, dC, dT_{ij} and for their measurement inaccuracies da, db, dc, dt_{ij} of all the loads used for the full two-port calibration we take the rather lengthy expressions,

$$\begin{aligned} L &= (ab + ct_{11})(B - A)(T_{11} - C) + (bc + at_{11})(C - B)(T_{11} - A) \\ &\quad + (ca + bt_{11})(A - C)(T_{11} - B) \\ &= [\textstyle\sum (ab + ct_{11})(B - A)(T_{11} - C)]/G \end{aligned} \tag{9}$$

$$\begin{aligned} T &= -(t_{21} - X)T_{12}(A - B)(a - b)(B - C)(b - c)(C - A)(c - a)]/GF \\ &= -(t_{21} - X)T_{12}[\textstyle\prod (A - B)(a - b)]/(GF) \end{aligned} \tag{10}$$

$$\begin{aligned} dL &= \{T_{12}T_{21}\textstyle\sum (B - C)(b - t_{11})(c - t_{11})[(B - C)(b - a)(c - a)dA \\ &\quad - (b - c)(B - A)(C - A)da] + T_{12}T_{21}[\textstyle\prod (A - B)(a - b)]\, dt_{11} \\ &\quad - T_{12}T_{21}E^2dT_{11} - H^2dT_{22} + EH(T_{21}dT_{12} + T_{12}dT_{21})\}/G^2 \\ &\quad = (1/G^2)\{T_{12}T_{21}((B - C)(b - t_{11})(c - t_{11}) \\ &\quad \cdot [(B - C)(b - a)(c - a)dA - (b - c)(B - A)(C - A)da] \\ &\quad + (C - A)(c - t_{11})(a - t_{11}) \\ &\quad \cdot [(C - A)(c - b)(a - b)dB - (c - a)(C - B)(A - B)db] \\ &\quad + (A - B)(a - t_{11})(b - t_{11}) \\ &\quad \cdot [(A - B)(a - c)(b - c)dC - (a - b)(A - C)(B - C)dc]) \\ &\quad + T_{12}T_{21}[(A - B)(a - b)(B - C)(b - c)(C - A)(c - a)]\, dt_{11} \\ &\quad - T_{12}T_{21}E^2dT_{11} - H^2dT_{22} + EH(T_{21}dT_{12} + T_{12}dT_{21})\} \end{aligned} \tag{11}$$

$$\begin{aligned} dT &= \{(t_{21} - X)T_{12}\textstyle\sum \{(A - B)(C - A)(a - b)(c - a)(T_{22}BC(b - c) \\ &\quad + \Delta[B(t_{11} - b) - C(t_{11} - c)]) - G[F + 2(a - b)B(A - C)]\} \\ &\quad \cdot \{[\textstyle\prod (a - b)](B - C)^2dA - [\textstyle\prod (A - B)](b - c)^2da\}/(F^2G^2) \end{aligned}$$

$$
\begin{aligned}
&+ [\prod (A - B)(a - b)]\{(t_{21} - X)[T_{12}(Idt_{11} + E(T_{22}dT_{11} - T_{12}dT_{21}) \\
&+ HdT_{22}) + HT_{22}dT_{12}]/G + T_{12}(dX - dt_{21})\}/(FG) \\
&= \{(t_{21} - X)T_{12}\{(A - B)(C - A)(a - b)(c - a)(T_{22}BC(b - c) \\
&+ \Delta[B(t_{11} - b) - C(t_{11} - c)]) - G[F + 2(a - b)B(A - C)]\} \\
&\cdot \{(a - b)(b - c)(c - a)(B - C)^2 dA \\
&- (A - B)(B - C)(C - A)(b - c)^2 da\} \\
&+ \{(B - C)(A - B)(b - c)(a - b)(T_{22}CA(c - a) \\
&+ \Delta[C(t_{11} - c) - A(t_{11} - a)]) - G[F + 2(b - c)C(B - A)]\} \\
&\cdot \{(a - b)(b - c)(c - a)(C - A)^2 dB \\
&- (A - B)(B - C)(C - A)(c - a)^2 db\} \\
&+ \{(C - A)(B - C)(c - a)(b - c)(T_{22}AB(a - b) \\
&+ \Delta[A(t_{11} - a) - B(t_{11} - b)]) - G[F + 2(c - a)A(C - B)]\} \\
&\cdot \{(a - b)(b - c)(c - a)(A - B)^2 dC \\
&- (A - B)(B - C)(C - A)(a - b)^2 dc\}\}/(F^2G^2) \\
&+ [(A - B)(a - b)(B - C)(b - c)(C - A)(c - a)] \\
&\cdot \{(t_{21} - X)[T_{12}(Idt_{11} + E(T_{22}dT_{11} - T_{12}dT_{21}) + HdT_{22}) + HT_{22}dT_{12}]/G \\
&+ T_{12}(dX - dt_{21})\}/(FG) \qquad (12)
\end{aligned}
$$

with

$$
\begin{aligned}
E &= (ab + ct_{11})(B - A) + (bc + at_{11})(C - B) + (ca + bt_{11})(A - C) \\
&= \sum (ab + ct_{11})(B - A) \qquad (13)
\end{aligned}
$$

$$
F = cC(B - A) + aA(C - B) + bB(A - C) = \sum cC(B - A) \qquad (14)
$$

$$
\begin{aligned}
G &= (\Delta - T_{22}C)(ab + ct_{11})(B - A) + (\Delta - T_{22}A)(bc + at_{11})(C - B) \\
&+ (\Delta - T_{22}B)(ca + bt_{11})(A - C) = \sum (\Delta - T_{22}C)(ab + ct_{11})(B - A) \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
H &= (ab + ct_{11})(B - A)(T_{11} - C) + (bc + at_{11})(C - B)(T_{11} - A) \\
&+ (ca + bt_{11})(A - C)(T_{11} - B) = \sum (ab + ct_{11})(B - A)(T_{11} - C) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
I &= (\Delta - T_{22}C)c(B - A) + (\Delta - T_{22}A)a(C - B) + (\Delta - T_{22}B)b(A - C) \\
&= \sum (\Delta - T_{22}C)c(B - A) \qquad (17)
\end{aligned}
$$

where ∑ and ∏ produce, obviously from the above relations, two more terms, from the given one, by cyclic rotation of the letters a, b, c and A, B, C respectively. The corresponding L', T', dL', dT' for the reverse direction resulted from (9), (17) respectively by similar expressions, with the replacement of a, b, c and A, B, C with the primed ones and the subscripts 11, 12, 21 and 22 with 22, 21, 12 and 11 respectively, in all of their occurrences in t_{ij} or T_{ij}. The expressions (4)-(17) were mechanically cross verified using a developed software program for symbolic computations.

Tab. 1 contains the comparison between a SLOdT full two-port calibration when a Direct/Zero Length Through is possible and a SLOT full two-port calibration when a Non-Zero Length Through is inevitable. The number of measurements and their inaccuracies remains the same in both schemes as well as the number of standards and their uncertainties for the corresponding one port (forward and reverse direction) calibration but one more two-port load is added, the Through, which introduces four (4) more uncertainties. Therefore, each S-parameter has finally a total differential error dS expressed in terms of 26 independent complex differentials instead of 22, that is in terms of 104 independent real quantities (52 real parameters and 52 real differential errors) instead of 88 in the case of the SLOdT, that is 16 more (8 + 8). Thus, the contour of the complex DERs result from 22 orthogonals and 4 circles and have 176 vertices if standard matching Loads are used.

A Non-Zero Length Through

A Through standard with Non-Zero Length could be either an adapter/barrel or a short segment of a transmission line. Then, T_{11} and T_{22} have zero value and the L, T, dL, dT from (4)-(7), are simplified as:

$$L = (t_{11} - D)/\{T_{12}T_{21}[R + M(t_{11} - D)]\} \quad (18)$$

$$T = (t_{21} - X)R/\{T_{21}[R + M(t_{11} - D)]\} \quad (19)$$

$$\begin{aligned} dL = -\{&[(1 - T_{12}T_{21}LM)(dD - dt_{11}) + T_{12}T_{21}(t_{11} - D)dM + T_{12}T_{21}LdR \\ &+ [R + M(t_{11} - D)]dT_{11} + L(t_{11} - D)dT_{22} + L[R + M(t_{11} - D)] \\ &\cdot (T_{21}dT_{12} + T_{12}dT_{21})\}/\{T_{12}T_{21}[R + M(t_{11} - D)]\} \end{aligned} \quad (20)$$

$$dT = \{T_{12}T_{21}TM(dD - dt_{11}) - T_{12}T_{21}T(t_{11} - D)dM + [(t_{21} - X)T_{12} - T_{12}T_{21}T]dR - T[R + M(t_{11} - D)]T_{12}dT_{21} - \{R(t_{21} - X) - T[R + M(t_{11} - D)]T_{21}\}dT_{12} - T(t_{11} - D)dT_{22} + RT_{12}(dt_{21} - dX)\}/\{T_{12}T_{21}[R + M(t_{11} - D)]\} \quad (21)$$

Tab. 1: Full Two-Port Calibration

Zero-Length Through	Non-Zero Length Through
16 Measurements	⇒ 16 Inaccuracies
6 Standard Loads	⇒ 6 Uncertainties
	1 Through Load ⇒ 4 Uncertainties
22 Complex Variables	26
22 Complex Differential Errors	26
44 Real Parameters	52
44 Real Differential Errors	52
S-DER SLOdT	S-DER SLOT
20 Orthogonals + 2 Circles	22 Orthogonals + 4 Circles
DER Contour: 160 Vertices	DER Contour: 176 Vertices

It is important to be noted that if the simplified expressions (18) and (19) were used, then the two terms with the uncertainty in T_{11} and T_{22} parameters in L, and the two terms with the uncertainty in T_{12} and T_{22} parameters in T, shown in red color above, would be lost. Since, usually, the terms ii correspond to return loss, or SWR, at specifications, this loss would lead to an unreal reduction of the final uncertainty of the L, T system errors.

The corresponding final expressions are given by

$$L = [\sum (ab + ct_{11})(B - A)C)]/(T_{12}T_{21}E) \quad (22)$$

$$T = (t_{21} - X)[\prod (A - B)(a - b)]/(T_{21}EF) \tag{23}$$

$$\begin{aligned} dL = \{&\sum (B - C)(b - t_{11})(c - t_{11})[(B - C)(b - a)(c - a)dA \\ &- (b - c)(B - A)(C - A)da] + [\prod (A - B)(a - b)]dt_{11}\}/(T_{12}T_{21}E^2) \\ &- dT_{11}/(T_{12}T_{21}) - J^2 dT_{22}/(T_{12}T_{21}E)^2 \\ &- J(T_{21}dT_{12} + T_{12}dT_{21})/[(T_{12}T_{21})^2 E] \end{aligned} \tag{24}$$

$$\begin{aligned} dT = \{&(t_{21} - X)\sum \{(A - B)(C - A)(a - b)(c - a)[B(t_{11} - b) - C(t_{11} - c)]) \\ &- E[F + 2(a - b)B(A - C)]\} \\ &\cdot \{[\prod (a - b)](B - C)^2 dA - [\prod (A - B)](b - c)^2 da\}/(T_{21}F^2E^2) \\ &- [\prod (A - B)(a - b)]\{(t_{21} - X)[(\sum c(B - A))dt_{11}/(T_{21}FE^2) \\ &+ dT_{21}/((T_{21})^2 FE) + JdT_{22}/(T_{12}(T_{21})^2 FE^2))] \\ &+ (dX - dt_{21})/(T_{21}FE)\} \end{aligned} \tag{25}$$

where H from (16) has been transformed to J

$$J = -\sum (ab + ct_{11})(B - A)C \tag{26}$$

SLOT System Error Differentials and Uncertainties

When the Indirect Through is due to the same type/sex of connectors on both ports then the commonly used Short, matching Load and Open calibration standards will be the same for both SLO one-port calibration, while if different types of connectors are used on the two ports, the standards will be different. Thus, the SLO values are

$$A = A' = -1$$

$$B = B' = 0 \tag{27}$$

$$C = C' = 1$$

The next issue is to determine the values T_{ij} of the Non-Zero Length Through standard. Considering it as a transmission line segment of length ℓ, then

$$T_{11} = T_{22} = 0 \tag{28a}$$

$$T_{12} = T_{21} = e^{-\gamma\ell} \tag{28b}$$

where $\gamma = \alpha + i\,\beta$ is the well-known complex propagation coefficient. (27) and (28) are substituted in (22)-(25) to produce the following expressions for the corresponding system errors and their differentials

$$L = e^{2\gamma\ell}(a - c)(b - t_{11})/[t_{11}(a + c - 2b) + b(a + c) - 2ca] \quad (29)$$

$$T = e^{\gamma\ell}2(t_{21} - X)(a - b)(b - c)/[t_{11}(a + c - 2b) + b(a + c) - 2ca] \quad (30)$$

$$\begin{aligned} dL = e^{2\gamma\ell}\{&(b - t_{11})(c - t_{11})[(b - a)(c - a)dA + 2(b - c)da] \\ &+ 2(c - t_{11})(a - t_{11})[2(c - b)(a - b)dB + (c - a)db] \\ &+ (a - t_{11})(b - t_{11})[(a - c)(b - c)dC + 2(a - b)dc] \\ &+ 2(a - b)(b - c)(c - a)dt_{11}\}/[t_{11}(a + c - 2b) + b(a + c) - 2ca]^2 \\ &- e^{2\gamma\ell}dT_{11} - e^{4\gamma\ell}(a - c)^2(b - t_{11})^2dT_{22}/[t_{11}(a + c - 2b) \\ &+ b(a + c) - 2ca]^2 - e^{2\gamma\ell}(a - c)(b - t_{11})(dT_{12} + dT_{21}) \\ &/ [t_{11}(a + c - 2b) + b(a + c) - 2ca] \end{aligned} \quad (31)$$

$$\begin{aligned} dT = e^{\gamma\ell}(t_{21} - X)\{&(c - a)^2(b - c)(b - t_{11}) \\ &\cdot [(b - a)(c - a)dA + 2(b - c)da] \\ &+ 2(c - a)[(t_{11} - a)(c - b)^2 - (c - t_{11})(a - b)^2] \\ &\cdot [2(c - b)(a - b)dB + (c - a)db] \\ &+ (a - b)(b - t_{11})(a - c)^2 \\ &\cdot [(a - c)(b - c)dC + 2(a - b)dc]\} \\ &/ [t_{11}(a + c - 2b) + b(a + c) - 2ca]^2(c - a)^2 \\ &+ e^{\gamma\ell}(t_{21} - X)2(a - b)(b - c)\{(2b - a - c)dt_{11} \\ &/ [t_{11}(a + c - 2b) + b(a + c) - 2ca] - e^{\gamma\ell}dT_{21} \\ &- e^{2\gamma\ell}(a - c)(b - t_{11})dT_{22} - dX + dt_{21}\} \\ &/ [t_{11}(a + c - 2b) + b(a + c) - 2ca] \end{aligned} \quad (32)$$

The red color in (29)-(32) indicates the additional factors and terms with respect to the Direct Through [1]. This implies that if we consider $T_{12} = T_{21} = 1$ and $dT_{ij} = 0$ then the result expressions will be identical with those for the Direct Through.

The remaining SLOT system errors and their differentials have already been expressed in [1]-[4].

For a lossless transmission line in high frequencies $\alpha\ell \approx 0$, $\beta\ell = 2\pi(\ell/\lambda)$ and $\lambda = cv_f/f$, with c the velocity of light, v_f the velocity factor and f the frequency. Thus,

$$\beta\ell = 2\pi\,\ell\, f[\mathrm{MHz}]/(v_f 300) \qquad (33)$$

$$\gamma\ell \approx i\,\beta\ell \qquad (34)$$

and from (28b)

$$T_{12} = T_{21} = e^{-i\beta\ell} \qquad (35)$$

The L, T, L', T' system errors are shown in Fig. 3 against frequency for the Direct Through and in Fig. 4 for the Indirect Through. Tab. 2 contains the standard loads uncertainties that were adopted for the present results, based on available manufacturer's data [3], [5]-[7]. Since T_{11} and T_{22} have nominal value zero their uncertainties dT_{11} and dT_{22} are represented by circular DERs with radius to be determined from the return loss (or SWR) specifications. On the other hand dT_{12} and dT_{21} are rectangular DERs. For their magnitude and phase uncertainties the specifications for insertion loss and electrical length were used from the most relevant reference since there were no available such data for the used Through. The length uncertainty is consistent with the mechanical tolerance. As measurement inaccuracies the ±1 digit in LSD for magnitude and phase was assumed as in [1]-[4].

Fig. 5 shows the L-DERs and DEIs at the frequency of 639 [MHz] where ZT stands for the Zero Length Through case (black), NZT for the Non-Zero Length Through case (blue), while the red colored DER corresponds to the NZT but without (w/o) take into account the Through standard in calculations and the green colored DER to the NZT but without the dT_{ii} terms. The T-DERs and DEIs are shown in Fig. 6 using the same notations. Although the L-DERs intersect in large areas the same is not true for T-DERs where in each case the corresponding differential error region (and intervals) is in a different scale, resulting in an impossible direct comparison as shown in Fig. 7(a). Thus, the regions are given in separate figures (b, c and d) from which their totally different shapes are depicted.

Tab. 2: Standard Loads Uncertainty

A, C		B	T_{11}, T_{22}	T_{12}, T_{21}	
–0.01, 0	±2°	0.029	0.025	±0.08[dB]	±0.0002[m]

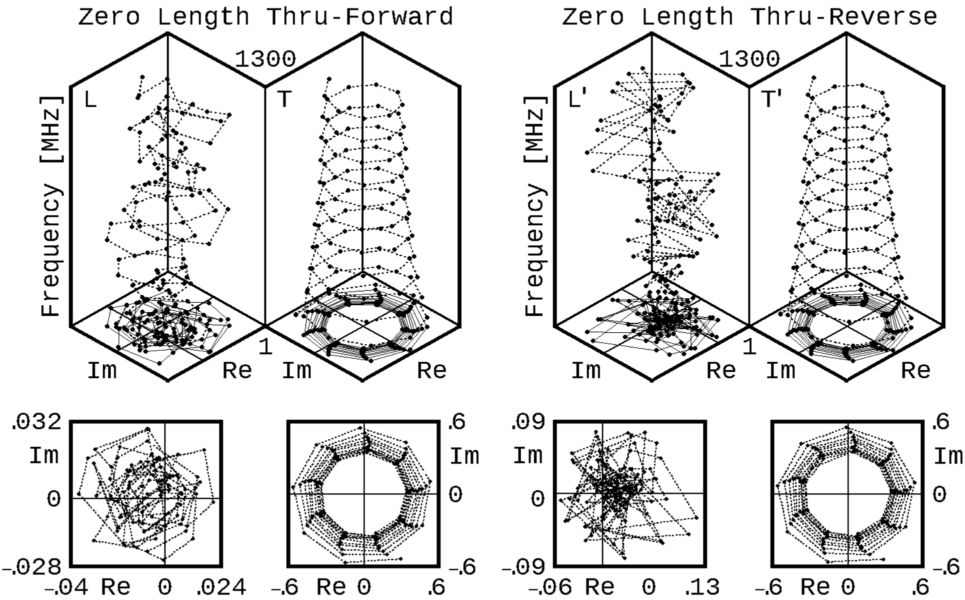

Fig. 3: L, T, L' T' for Zero-Length Through

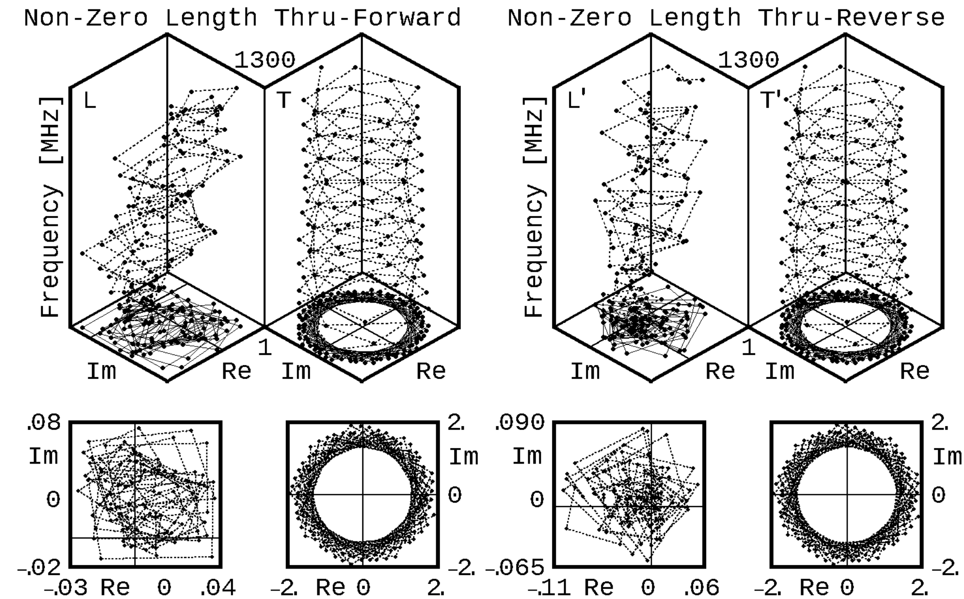

Fig. 4: L, T, L' T' for Non-Zero Length Through

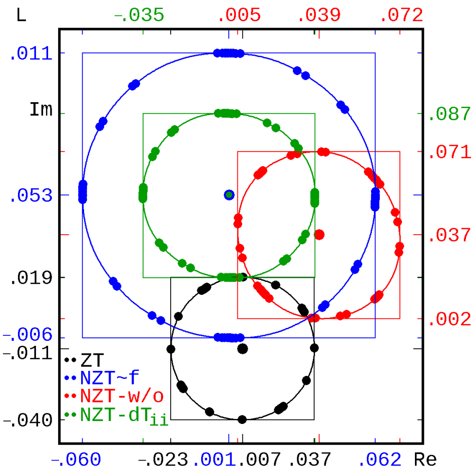

Fig. 5: L-DERs at 639 [MHz]

Results

The two-port DUT is shown in Fig. 7. It is a simple T-resistive network with input impedance far from 50 Ohms for both ports, with horizontal arms Z_1 = 24.2 [Ω], Z_2 = 120 [Ω] and vertical arm Z_{12} = 1.1 [Ω]. This box was constructed with female connector at both ports, and a female to female adapter served as the through standard for the SLOT calibration. Then the connector of port 1 was changed to male and the SLOdT calibration method was applied [1], [2]. The VNA system used was described in [1] and the measurements were made from 2 to 1289 [MHz].

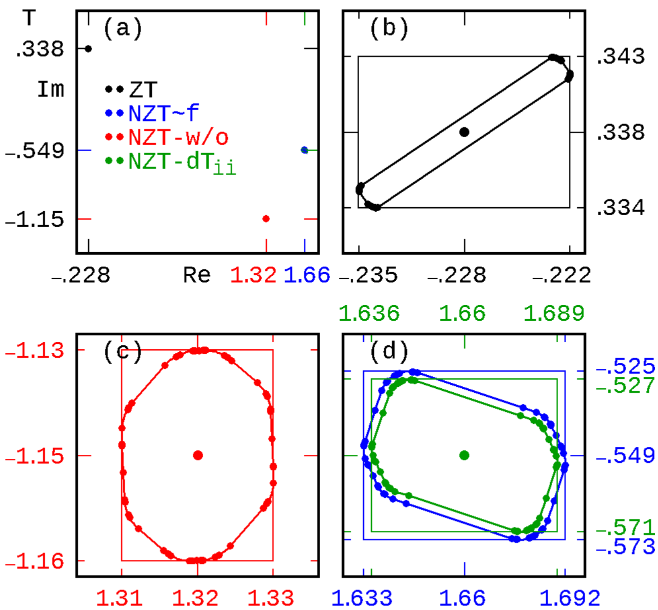

Fig. 6: T-DERs at 639 [MHz]

Fig. 7: The T-Resistive box in situ

The typical details for the Type-N connectors, and especially those for the reference plane were taken initially by [8]. A figure for both sexes with all the dimensions is illustrated there and there is also an indication of a plus/minus mechanical tolerance.

After a careful measurement of the real dimensions of the used adapter with a digital vernier caliper and from its specifications [5], its length ℓ was determined from its physical length of 47.48 [mm] and the reference plane of the female connectors, as

$$\ell = 47.48 - 2 \cdot 9 = 29.48 \text{ [mm]}$$

The adapter was then considered as a small segment of lossless transmission line with return loss 32 [dB] for its two ports (specifications before 2007). This piece of line can be considered as an "almost short" Through connection as it is equal to about one-tenth of a wavelength at the frequency, for example, of 1 [GHz] and not to one-hundredth as it is mentioned in [9]. Actually, it varies between 0.2 to 13 hundredths in the whole measured frequency range.

Fig. 8 shows analytically the measurements of S_{21} DUT parameter. At (8a) the black colored curve corresponds to the measurement of S_{21} with SLOdT (ZT: Zero Length Thru) and the red color curve to the measurements of S_{21} with a Non-Zero Length Through but without (w/o) take it into account in calculations (NZT: Non-Zero Length Thru). It is obvious, how the presence of the Through standard affected the measurements. At (8b) the green color curve corresponds to the measurements of S_{21} with Non-Zero Length Through but now, we do consider it, having an electrical length at 1 [GHz]. Below at (8c) the magenta color curve resulted for the Through with an electrical length at the central frequency of 639 [MHz] and at (8d) the blue colored curve for the Through with electrical length as function of frequency.

The middle frequency was selected since it is important for the determination of the reference plane if the TRL calibration will be applied while the 1 [GHz] was selected as a characteristic frequency, both for comparison reasons [9], [10].

It is essential that the most accurate measurement of SLOdT with the Direct Through connection was used as reference, that is, the black curve, and the conclusion was that a Non-Zero Length Through must be considered as frequency dependent.

In Fig. 9 all the four S parameters are shown. The four above mentioned cases are examined for the Through measurements both for Direct and Indirect connection for each S parameter. It is evident, how the presence of the Through standard affected the measurements.

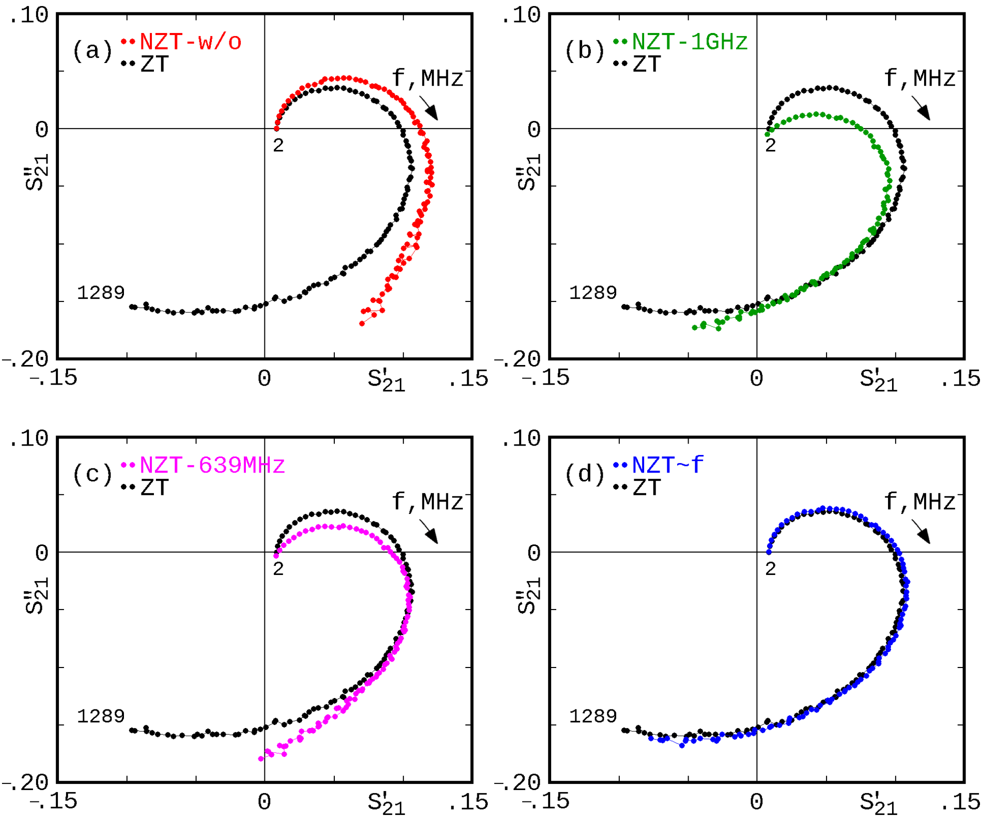

Fig. 8: S_{21} with SLOdT and SLOT

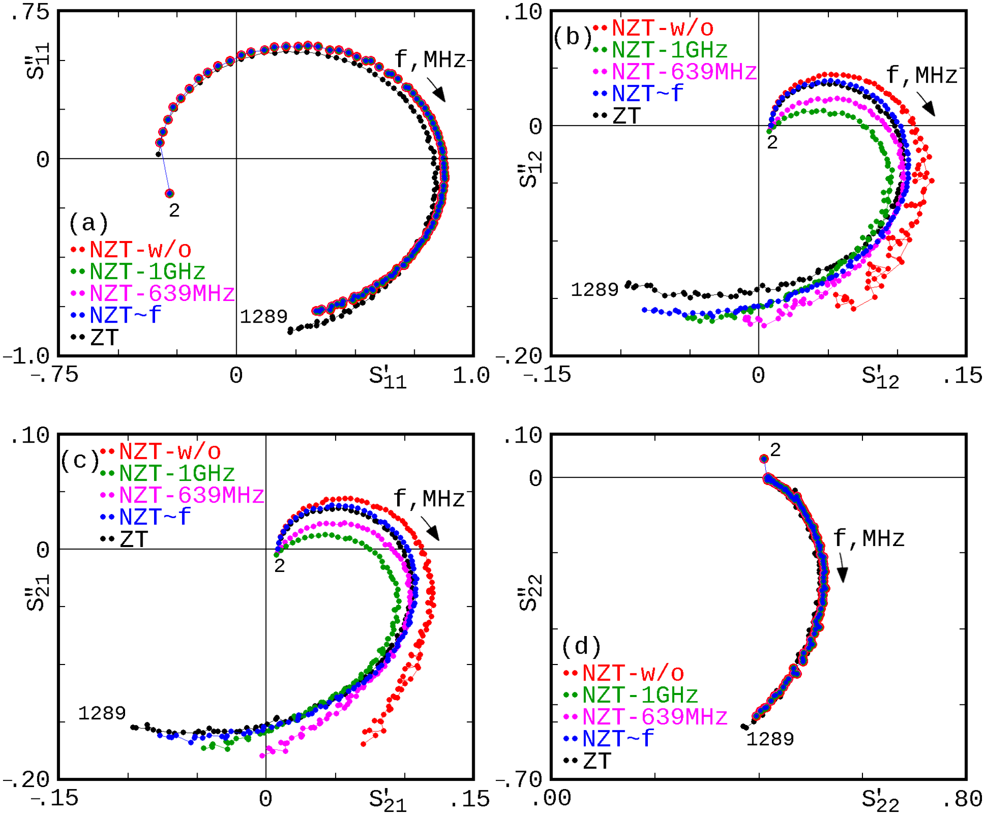

Fig. 9: [S] Matrix with SLOdT and SLOT

Obviously there is a very small change at S_{11} and S_{22} measurements, but the Through influence can not be omitted at S_{12} and S_{21}.

To illustrate the DERs for the four S parameters of the DUT the three most important cases have been chosen to be included: (i) the SLOdT results in black color, (ii) the Non-Zero Through is used but it is neglected, in red color, and (iii) the results from the full expressions for the differential errors when SLOT calibration is performed with a Non-Zero Length Through between the two ports which is not neglected, in blue color.

Fig. 10 shows the S_{11} parameter. A number of selected S-DER frames are drawn explicitly. At the last right figure we tried to give a comparison of these cases, which is rather difficult in the complex plane.

The shape of the DERs in Fig. 10 is different and variable with frequency. But since this is the S_{11} parameter we can see that DERs are overlapping almost for the entire frequency range for all the three cases

In Fig. 11 S_{21} parameter is given. The differences here are clearer. Obviously, the DERs of the SLOT case (blue color) are larger from those of SLOdT, as it was expected since four more uncertainties have been added. The same is true for the S_{12} parameter in Fig. 12.

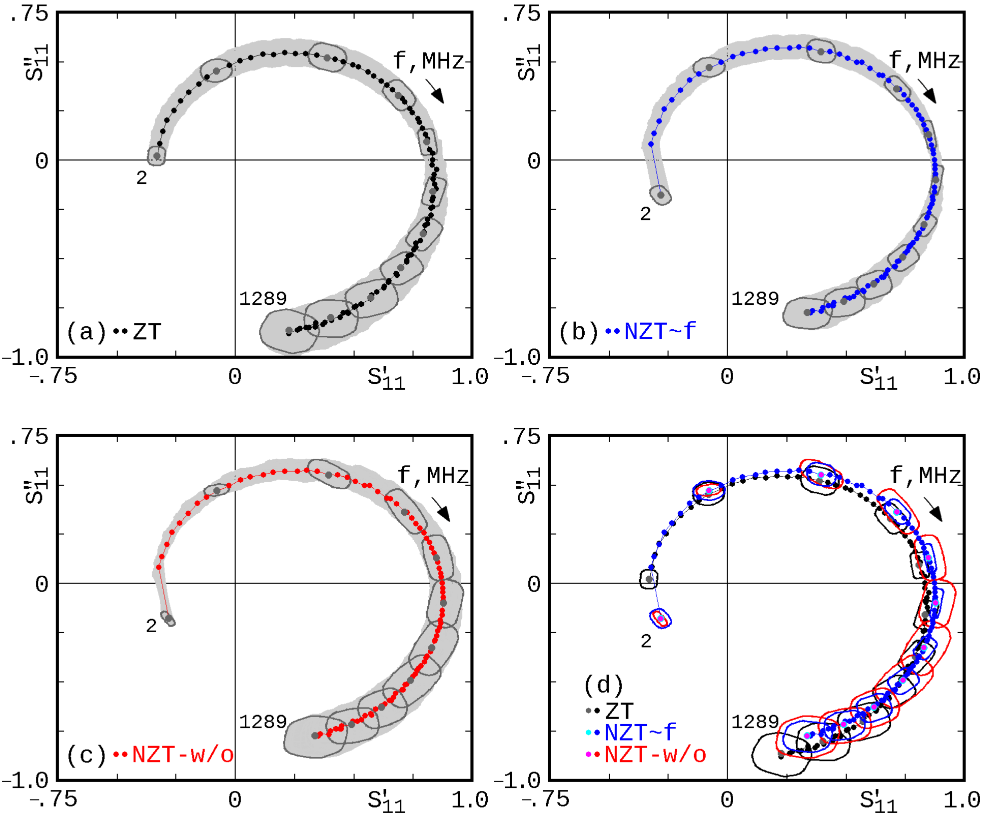

Fig. 10: S_{11}-DER with SLOdT and SLOT

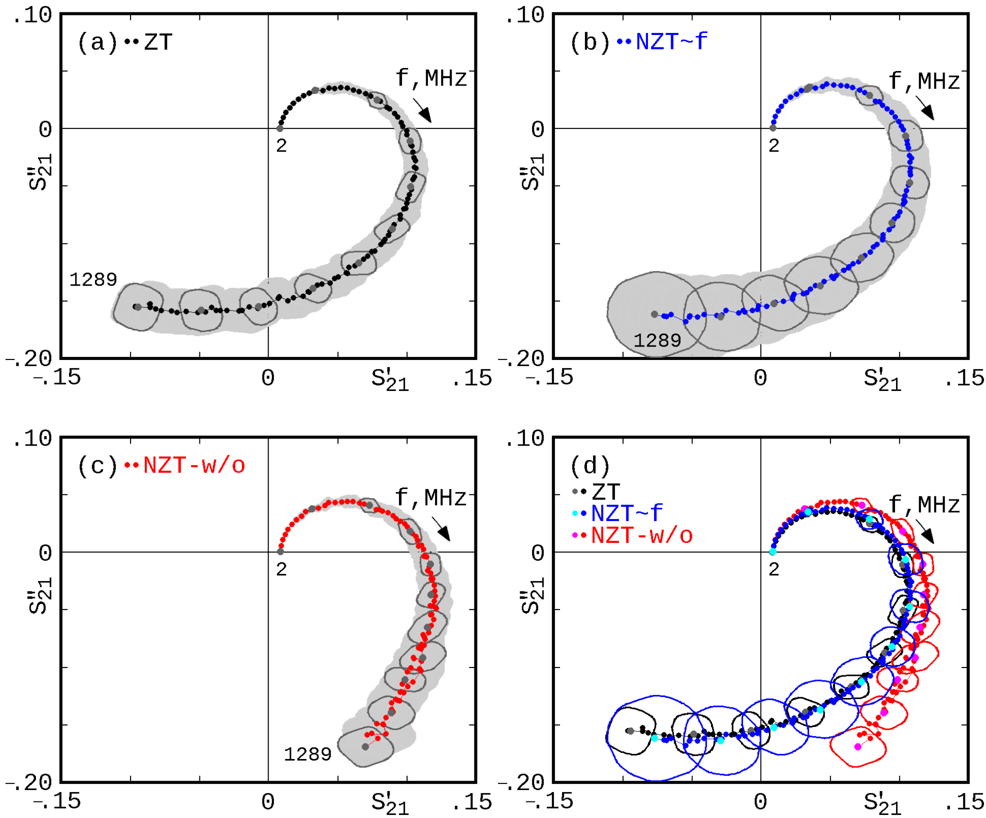

Fig. 11: S_{21}-DER with SLOdT and SLOT

The figures with the measurement results for S_{21} and S_{12} are almost overlapping, and the reciprocity is approximately satisfied for the DUT. A simple way to visualize that fact is to move back and forward between the two figures.

At Fig. 13 the S_{22}-DERs are shown. From the last right figure (d) it is obvious that there is almost none effect at the S_{22} by considering or not the influence of Non-Zero Length Through.

Fig. 14 shows the S_{11}-DERs for the three examined cases at the central frequency. The rectangular Differential Error Intervals [DEIs] (Real and Imaginary) and the polar DEIs (magnitude and phase) are included.

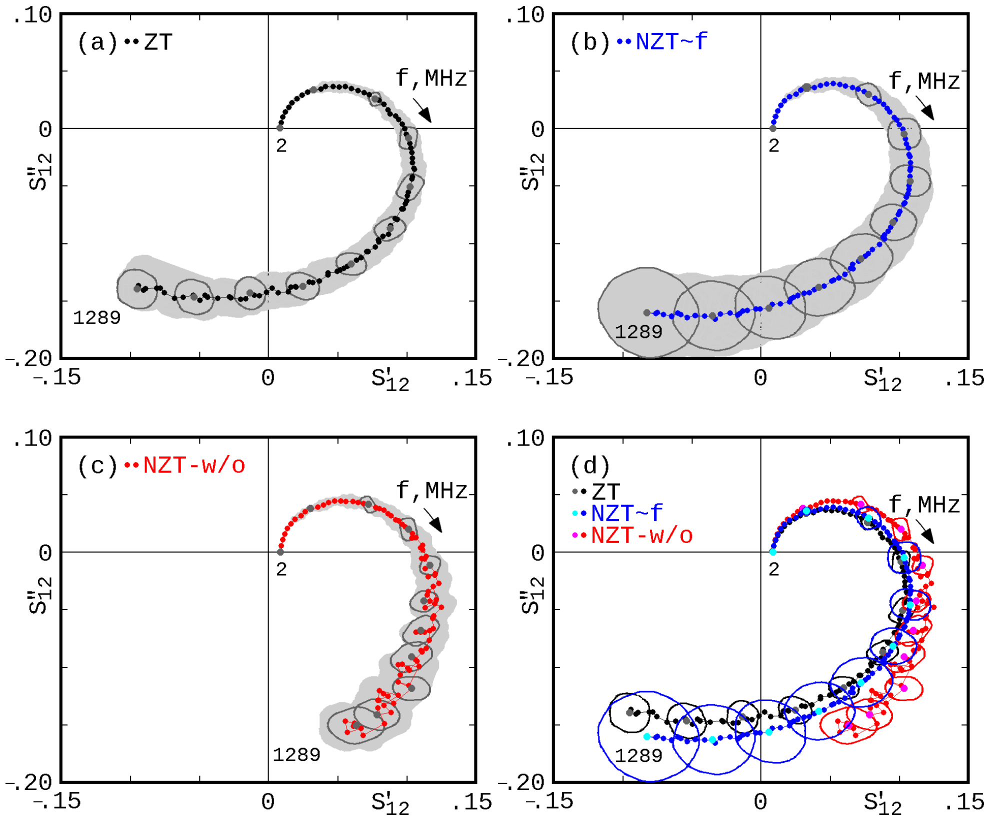

Fig. 12: S_{12}-DER with SLOdT and SLOT

The black DER in Fig. 14 (Direct Through), has a contour of 160 points, while the blue DER (Indirect Through) has a contour of 176 points. At this frequency, the DER for the case of not taking into account the presence of the Through (red color), is indeed much larger. It is worth to notice that S_{11} with the Non-Zero Length Through standard, blue point, is inside the black DER. A green colored DER that is slightly smaller than the blue DER of non-zero Through case, and hardly shown in the figure is the one that results from the full expressions for the Non -Zero Length Through but by considering that differential errors dT_{11}, dT_{22} are zero.

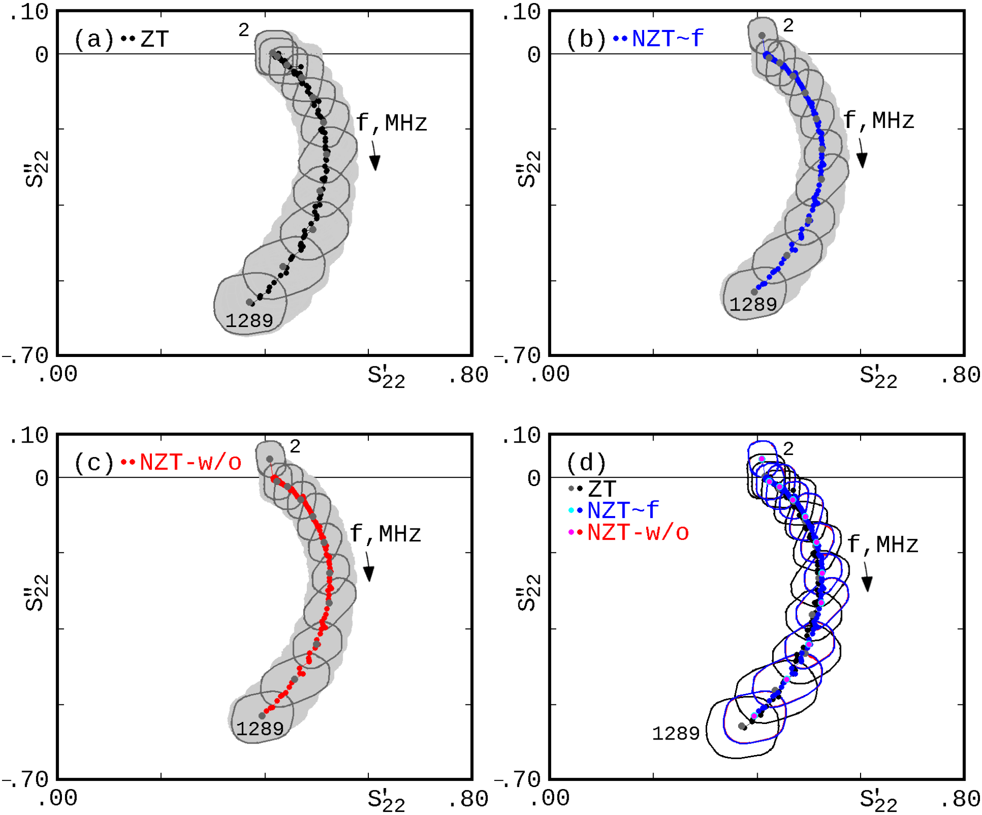

Fig. 13: S_{22}-DER with SLOdT and SLOT

In Fig. 15 the S_{21}-DERs with the corresponding Rectangular and Polar DEIs, are shown. The blue S_{21} point that corresponds to the Non-Zero Length Through measurements is again inside the black DER from the Zero Length Through case. Obviously there is an overlapping of these two DERs. The green DER is now well shown.

Since Figs. 14 and 15 are already too complicated, Tab. 3 contains the magnitude in [dB] and the phase in [°] of S_{11} and S_{21} in 639 [MHz] for the four illustrated polar DEIs with the corresponding signed maximum values (positive and negative, not symmetrical in general) of the estimated uncertainties.

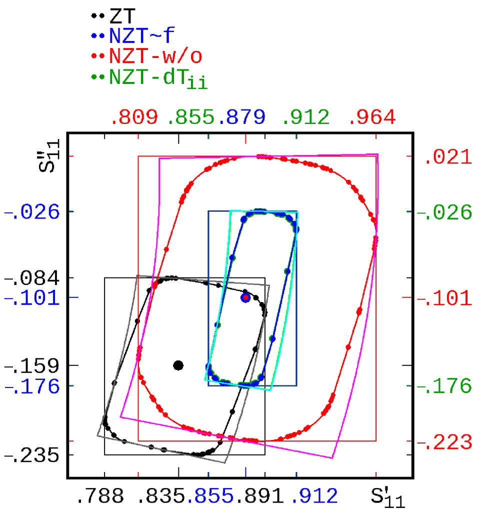

Fig. 14: S_{11}-DER and DEIs

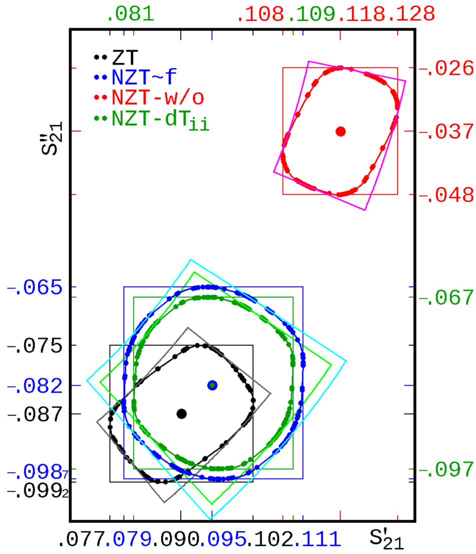

Fig. 15: S_{21}-DER and DEIs

<u>Tab. 3: Positive and Negative Maximum Uncertainty</u>

	S_{11}					
	Magnitude [dB]			Phase [°]		
	Value	–	+	Value	–	+
ZT	−1.408	0.393	0.481	−10.75	4.87	5.00
NZT-w/o	−1.063	0.627	0.754	−6.52	7.73	7.88
NZT-dT_{ii}	−1.063	0.149	0.270	−6.53	4.81	4.83
NZT	−1.063	0.153	0.274	−6.53	4.84	4.85
	S_{21}					
ZT	−18.067	0.668	0.632	−44.23	5.71	5.80
NZT-w/o	−18.174	0.634	0.599	−17.44	5.22	5.19
NZT-dT_{ii}	−18.023	1.089	0.951	−40.85	6.53	6.81
NZT	−18.023	1.231	1.062	−40.85	7.35	7.64

Conclusion

The most complex case of the full two-port calibration when a Non-Zero Length standard as Through connection is required was examined analytically. The most complicated expressions where simplified and applied in the case of SLOT calibration on a simple T-resistive network.

In contrast with SLO standards, the parameters value of an Indirect Through is frequency depended and its influence cannot be neglected especially for the transmission scattering parameters.

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